

32. Both \vec{r} and \vec{v} lie in the xy plane. The position vector \vec{r} has an x component that is a function of time (being the integral of the x component of velocity, which is itself time-dependent) and a y component that is constant ($y = -2.0$ m). In the cross product $\vec{r} \times \vec{v}$, all that matters is the y component of \vec{r} since $v_x \neq 0$ but $v_y = 0$:

$$\vec{r} \times \vec{v} = -yv_x \hat{k} .$$

- (a) The angular momentum is $\vec{\ell} = m(\vec{r} \times \vec{v})$ where the mass is $m = 2.0$ kg in this case. With SI units understood and using the above cross-product expression, we have

$$\vec{\ell} = (2.0) (-(-2.0) (-6.0t^2)) \hat{k} = -24t^2 \hat{k}$$

in $\text{kg}\cdot\text{m}^2/\text{s}$. This implies the particle is moving clockwise (as observed by someone on the $+z$ axis) for $t > 0$.

- (b) The torque is caused by the (net) force $\vec{F} = m\vec{a}$ where

$$\vec{a} = \frac{d\vec{v}}{dt} = -12t \hat{i} \text{ m/s}^2 .$$

The remark above that only the y component of \vec{r} still applies, since $a_y = 0$. We use $\vec{\tau} = \vec{r} \times \vec{F} = m(\vec{r} \times \vec{a})$ and obtain

$$\vec{\tau} = (2.0) (-(-2.0)(-12t)) \hat{k} = -48t \hat{k}$$

in N·m. The torque on the particle (as observed by someone on the $+z$ axis) is clockwise, causing the particle motion (which was clockwise to begin with) to increase.

- (c) We replace \vec{r} with \vec{r}' (measured relative to the new reference point) and note (again) that only its y component matters in these calculations. Thus, with $y' = -2.0 - (-3.0) = 1.0$ m, we find

$$\vec{\ell}' = (2.0) (-(1.0) (-6.0t^2)) \hat{k} = 12t^2 \hat{k}$$

in $\text{kg}\cdot\text{m}^2/\text{s}$. The fact that this is positive implies that the particle is moving counterclockwise relative to the new reference point.

- (d) Using $\vec{\tau}' = \vec{r}' \times \vec{F} = m(\vec{r}' \times \vec{a})$, we obtain

$$\vec{\tau}' = (2.0) (-(1.0) (-12t)) \hat{k} = 24t \hat{k}$$

in N·m. The torque on the particle (as observed by someone on the $+z$ axis) is counterclockwise, relative to the new reference point.